

NAG Toolbox for MATLAB

f07mu

1 Purpose

f07mu estimates the condition number of a complex Hermitian indefinite matrix A , where A has been factorized by f07mr.

2 Syntax

```
[rcond, info] = f07mu(uplo, a, ipiv, anorm, 'n', n)
```

3 Description

f07mu estimates the condition number (in the 1-norm) of a complex Hermitian indefinite matrix A :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1.$$

Since A is Hermitian, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

4 References

Higham N J 1988 FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – **string**

Indicates how A has been factorized.

uplo = 'U'

$A = PUDU^H P^T$, where U is upper triangular.

uplo = 'L'

$A = PLDL^H P^T$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **a(lda,*)** – **complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

Details of the factorization of A , as returned by f07mr.

3: **ipiv(*)** – **int32 array**

Note: the dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

Details of the interchanges and the block structure of D , as returned by f07mr.

4: **anorm** – double scalar

The 1-norm of the **original** matrix A , which may be computed by calling Missing 'id'. **anorm** must be computed either **before** calling f07mr or else from a copy of the original matrix A .

Constraint: **anorm** ≥ 0.0 .

5.2 Optional Input Parameters1: **n** – int32 scalar

Default: The second dimension of the array **a** The dimension of the array **ipiv**.

n , the order of the matrix A .

Constraint: **n** ≥ 0 .

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work

5.4 Output Parameters1: **rcond** – double scalar

An estimate of the reciprocal of the condition number of A . **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*, A is singular to working precision.

2: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **a**, 4: **lda**, 5: **ipiv**, 6: **anorm**, 7: **rcond**, 8: **work**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to f07mu involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 5 and never more than 11. Each solution involves approximately $8n^2$ real floating-point operations but takes considerably longer than a call to f07ms with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The real analogue of this function is f07mg.

9 Example

```
uplo = 'L';  
a = [complex(-1.36, +0), complex(0, +0), complex(0, 0), complex(0, 0);  
      complex(1.58, -0.9), complex(-8.87, 0), complex(0, 0), complex(0,  
0);  
      complex(2.21, +0.21), complex(-1.84, +0.03), complex(-4.63, +0),  
complex(0, 0);  
      complex(3.91, -1.5), complex(-1.78, -1.18), complex(0.11, -0.11),  
complex(-1.84, +0)];  
anorm = 14.6641984095488;  
[a, ipiv, info] = f07mr(uplo, a);  
[rcond, info] = f07mu(uplo, a, ipiv, anorm)
```

```
rcond =  
      0.1497  
info =  
      0
```